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## ABSTRACT

The purpose of this study was to determine whether a process model could be constructed using steps identified from flow charts which accounted for somewhat more variance in predicting the difficulty of two-digit multiplication problems than did a process model developed by Cromer. Cromer's data and variables were used as a starting point. Ten new variables were identified from multiplication and addition flow charts. Seven basic models, 4 reduced models, 10 factor models, 24 one-variable models, and a set of systematic restricted models were examined. Multiple regression analysis was used to predict difficulty. The overall results indicate that the flow chart variables do produce somewhat better models. This volume presents the first of two parts of this report and includes the problem statement and results. (Author/SD)

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(PART 1 OF 2 PARTS)

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JULY 1975

WISCONSIN RESEARCH  
AND DEVELOPMENT  
CENTER FOR  
COGNITIVE LEARNING

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Technical Report No. 337 (Part 1 of 2 Parts)

PROCESS MODELS FOR PREDICTING THE DIFFICULTY  
OF MULTIPLICATION PROBLEMS USING FLOW CHARTS

by

Thomas A. Romberg  
Richard Glove

Report from the Project on  
Conditions of School Learning and  
Instructional Strategies

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## ABSTRACT

The purpose of this study was to determine whether a process model could be constructed using steps identified from flow charts which accounted for somewhat more variance in predicting the difficulty of two-digit multiplication problems than did a process model developed by Cromer (1971). Cromer's data and variables were used as a starting point. Ten new variables were identified from multiplication and addition flow charts. Seven basic models, 4 reduced models, 10 factor models, 24 one-variable models, and a set of systematic restricted models were examined. Multiple regression analysis was used to predict difficulty. The overall results indicate that the flow chart variables do produce somewhat better models.

I

## THE PROBLEM AND MODELS

### INTRODUCTION

While endeavoring to build an adequate theory of mathematics learning, some mathematics educators have attempted to construct and refine theoretical models of mathematics learning confined to specific areas of mathematics. One type of model that has been studied is the process model for the learning of an arithmetic skill.

The purpose of this study was to refine such a model. More work on models needs to be done in order to construct a reasonable theory of mathematics learning which will help teachers to give better instruction to their students in all areas of mathematics.

### VARIABLES

Fred Cromer (1971) constructed a model for predicting the difficulty of two-digit by two-digit multiplication problems. In his model, Cromer considered problems of the form

$$\begin{array}{r} ab \\ \times cd \\ \hline \end{array}$$

where a, b, c, and d are digits, with a and c representing the tens digits and b and d the units digits. Any of a, b, c, or d could be equal to 0. If c = 0, the problem was written

$$\begin{array}{r} ab \\ \times d \\ \hline \end{array}$$

If a = 0 and c = 0, the problem was written

$$\begin{array}{r} b \\ \times d \\ \hline \end{array}$$

No problems of the form

$$\begin{array}{r} b \\ \times cd \\ \hline \end{array}$$

were considered, since the students had not had exposure to this type of problem. In attempting to predict the difficulties of these problems, Cromer used 14 variables.

ORD--the order of the problem as presented on the test  
 TDF--the value of the tens digit of the first number  
 UDF--the value of the units digit of the first number  
 TDS--the value of the tens digit of the second number  
 UDS--the value of the units digit of the second number  
 OA--the number of operation steps in addition

OA was defined as follows:

$$OA(a + b) = \begin{cases} 0 & \text{if } a = 0 \text{ or } b = 0 \\ 1 & \text{otherwise} \end{cases}$$

$$OA(ab + d) = \begin{cases} OA(b + d) & \text{if } b + d \leq 9 \\ OA(b + d) + 1 & \text{if } b + d > 9 \end{cases}$$

$$OA(ab + cd) = \begin{cases} OA(b + d) + 1 & \text{if } b + d \leq 9 \\ OA(b + d) + 2 & \text{if } b + d > 9 \end{cases}$$

$$OA(abc + def) = OA(a + d) + OA(bc + ef) + K$$

$$\text{where } K = \begin{cases} 0 & \text{if } bc + ef \leq 99 \\ 1 & \text{if } bc + ef > 99 \end{cases}$$

$$OA(abcd + efg) = OA(a + e) + OA(bcd + fgh) + K$$

$$\text{where } K = \begin{cases} 0 & \text{if } bcd + fgh \leq 999 \\ 1 & \text{if } bcd + fgh > 999 \end{cases}$$

OM--the number of operation steps in multiplication

OM was defined as follows:

$$OM(a \times b) = \begin{cases} 0 & \text{if } a \text{ or } b \text{ equals 0 or 1} \\ 1 & \text{otherwise} \end{cases}$$

$$OM(ab \times c) = OM(a \times b) + OM(b \times c) + K$$

$$\text{where } K = \begin{cases} 0 & \text{if } b \times c \leq 9 \\ 1 & \text{if } b \times c > 9 \end{cases}$$

$$OM(ab \times cd) = OM(ab \times c) + OM(ab \times d)$$

DCA--the number of digits carried in addition

DCA was defined as follows:

$$\text{DCA}(abcd + efg) = K_1 + K_2 + K_3$$

where

$$K_1 = \begin{cases} 0 & \text{if } d + h \leq 9 \\ 1 & \text{if } d + h > 9 \end{cases}$$

$$K_2 = \begin{cases} 0 & \text{if } cd + gh \leq 99 \\ 1 & \text{if } cd + gh > 99 \end{cases}$$

$$K_3 = \begin{cases} 0 & \text{if } bcd + fgh \leq 999 \\ 1 & \text{if } bcd + fgh > 999 \end{cases}$$

DCM--the number of digits carried in multiplication

DCM was defined as follows:

$$\text{DCM}(ab \times cd) = \begin{cases} 0 & \text{if } b \times d \leq 9 \text{ and } b \times c \leq 9 \\ 1 & \text{if } b \times d > 9 \text{ or } b \times c > 9 \text{ but not both} \\ 2 & \text{if } b \times d > 9 \text{ and } b \times c > 9 \end{cases}$$

LDF--the largest digit in the factors

SDF--the smallest digit in the factors

NDP--the number of digits in the product

SMD--a variable to indicate whether the units and tens digits of either of the factors were the same

$$\text{SMD}(ab \times cd) = \begin{cases} 0 & \text{if } a \neq b \text{ and } c \neq d \\ 1 & \text{if } a = b \text{ or } c = d \end{cases}$$

LFS--a variable to indicate whether the second factor was larger than the first factor

$$\text{LFS}(ab \times cd) = \begin{cases} 0 & \text{if } ab < cd \\ 1 & \text{if } ab \geq cd \end{cases}$$

As the dependent variable, Cromer used a variable for the general problem difficulty (DIFF). He determined the value for this variable by administering two forms of an 84-problem multiplication test to 238 fifth-grade students. The problems were determined by using a random number routine. DIFF was defined to be the proportion of the students who failed to arrive at the correct solution. Thus, the lower the value of DIFF, the less difficult the problem was. The range of DIFF could be 0 to 1.

The purpose of this study was to consider several variations of Cromer's model that could be achieved by replacing some of his variables with other variables in the hope of arriving at a better prediction model. Thus, using Cromer's data as a basis, new models were constructed. Cromer's variables OM and OA failed to differentiate adequately between different types of problems. For example, the following problems all have OM = 2, using Cromer's definition:

$$\begin{array}{ccccc}
 42 & 40 & 61 & 82 & 15 \\
 \underline{x} & \underline{34} & \underline{55} & \underline{41} & \underline{20}
 \end{array}$$

A flow chart description of algorithms by Romberg and Anglin (1973) appeared to differentiate between more different classes of problems. The flow chart indicating the various steps that were involved in completing the multiplication part of a multiplication problem is shown in Figure 1.

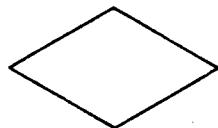
In the diagram:



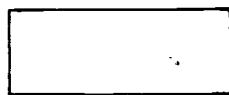
indicates that the procedure stops, starts, or goes to a different routine



information is entered into the procedure



a decision is made in the procedure



an operation is performed; something is computed or calculated



something is written

Because the different types of steps may differentially influence difficulty, it was decided to count the number of decisions made, the number of calculations done, and the number of writing operations involved in each problem, as well as the number of total steps needed to complete the routine. Therefore, the following variables were considered:

NDM--the number of decisions that an individual would have to make when going through the multiplication routine

YDM--the number of those decisions to which the answer yes was given

NOM--the number of operational steps that were involved in the multiplication routine

NWM--the number of writing steps in the multiplication routine

TSM--the total number of steps in the multiplication routine

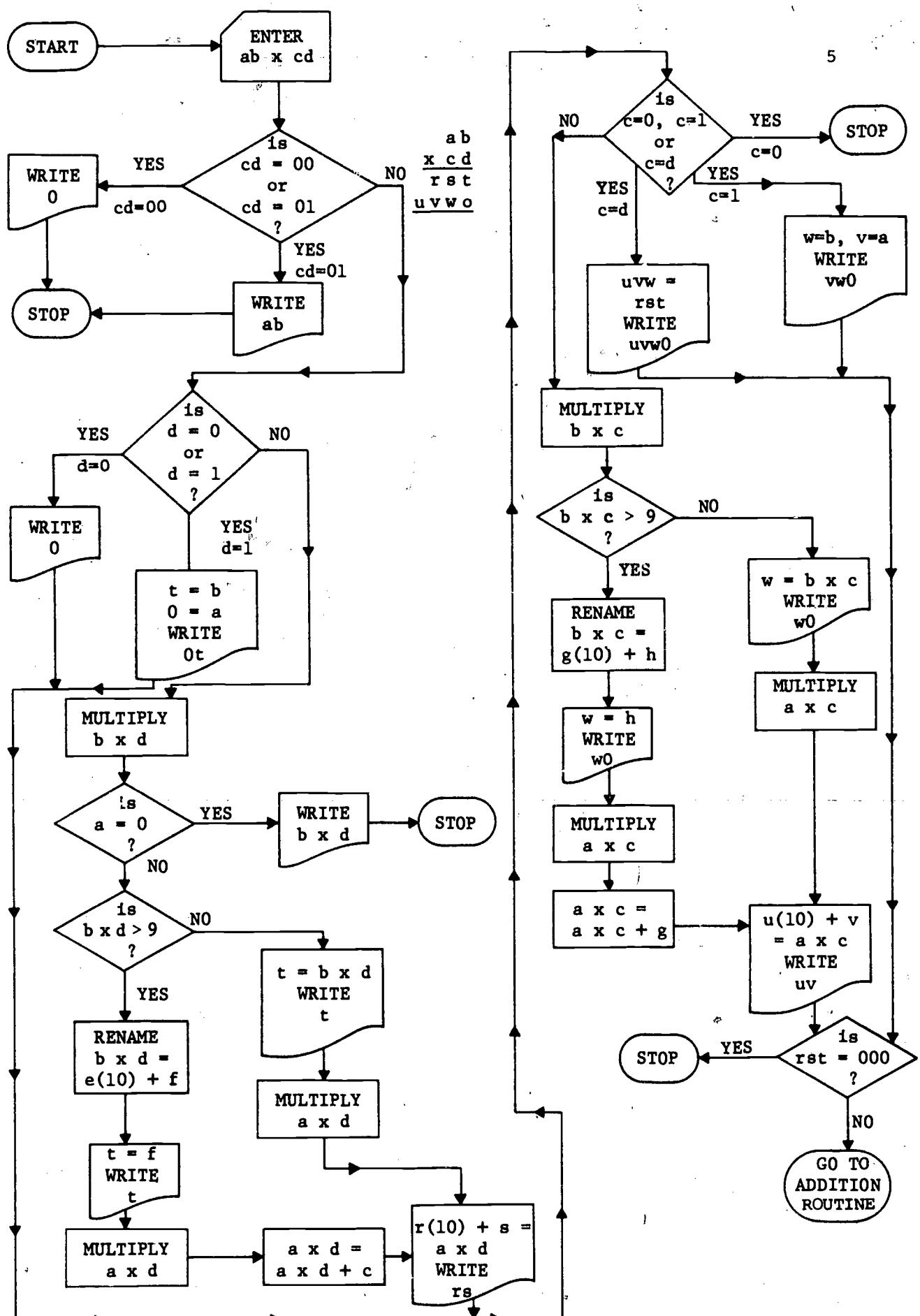


Figure 1. Flow chart for the multiplication part of a multiplication problem.

So, for the problems indicated above, where Cromer's variable OM = 2, the values for these new variables are:

	<u>42</u> <u>x 2</u>	<u>40</u> <u>x34</u>	<u>61</u> <u>x55</u>	<u>82</u> <u>x41</u>	<u>15</u> <u>x20</u>
NDM =	5	7	6	5	5
YDM =	1	0	1	1	3
NOM =	2	4	2	2	4
NWM =	2	4	3	3	3
TSM =	9	15	11	10	12

It should be noted that NDM + NOM + TSM. The other types of steps involved in the routine were not considered, since their values would not have differed for the problems considered and since the problems were already printed for the student to read and use.

Similarly, a flow chart was constructed for the addition part of a multiplication problem (see Figure 2). The following variables were defined for the addition routine.

NDA--the number of decisions that an individual would have to make when going through the addition routine

YDA--the number of those decisions to which the answer yes was given

NOA--the number of operational steps that were involved in the addition routine

NWA--the number of writing steps in the addition routine

TSA--the total number of steps in the addition routine

Here also, NDA + NOA + NWA = TSA.

The values of these 10 new variables and the values of Cromer's 15 variables for all 168 problems are given in Table 58 in Appendix A.

## MODELS

Since a slightly different statistical package was available on the Univac 1110 at the University of Wisconsin-Madison from the one Cromer used, his original models were also re-evaluated so that comparisons could be made using the same procedure. The REGAN 2 program for multiple linear regression analysis (1971) was used to evaluate the models. The models examined fit into five categories: basic models, reduced models, factor models, one-variable models, and restricted models.

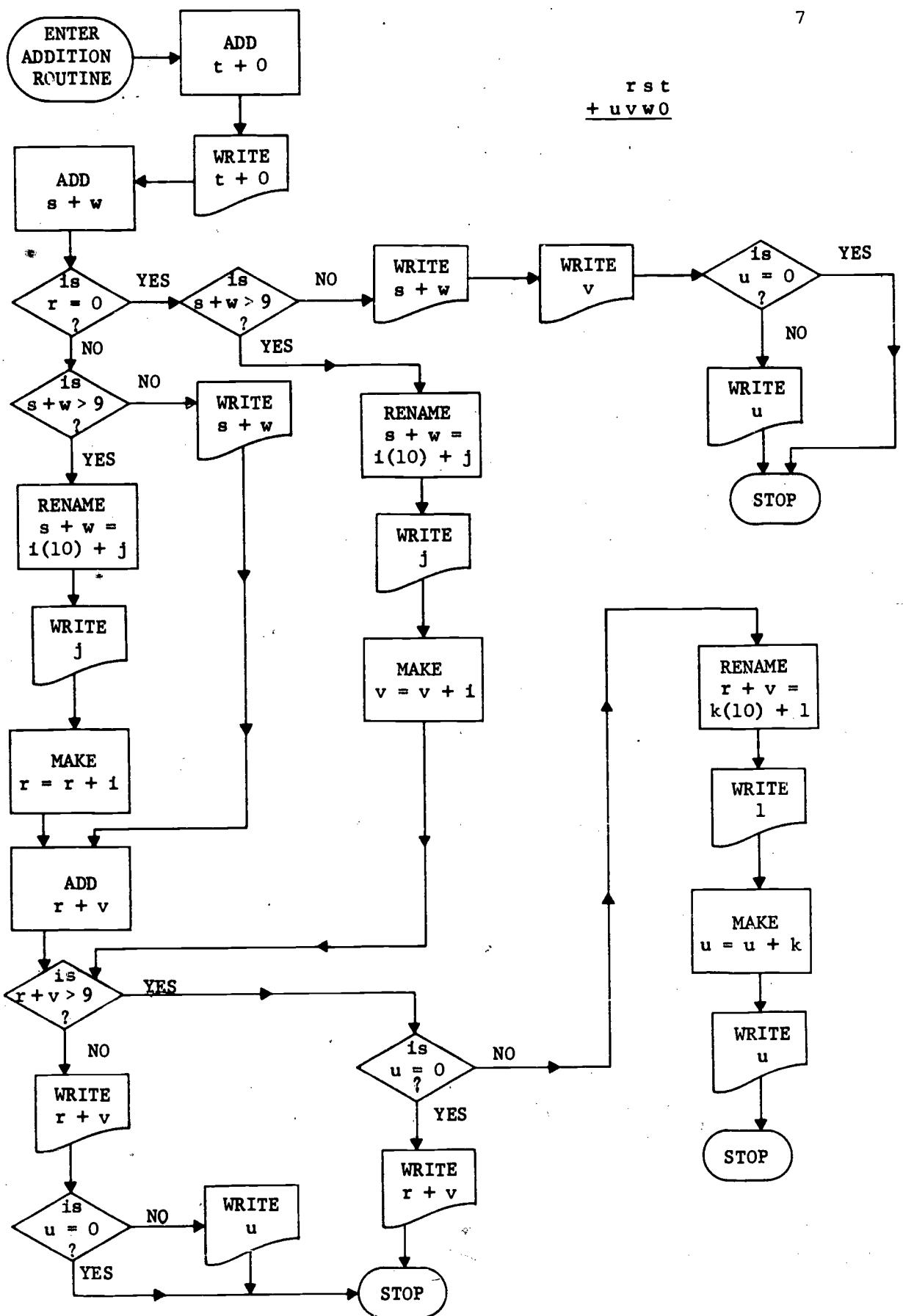


Figure 2. Addition flow chart.

### Basic Models

Model 1 (the complete Cromer model) consists of all of Cromer's 14 variables used as predictors.

$$\begin{aligned} \text{DIFF} = & A_0 + A_1 \cdot \text{ORD} + A_2 \cdot \text{TDF} + A_3 \cdot \text{UDF} + A_4 \cdot \text{TDS} + A_5 \cdot \text{UDS} + \\ & A_{16} \cdot \text{OA} + A_{17} \cdot \text{OM} + A_{18} \cdot \text{DCA} + A_{19} \cdot \text{DCM} + A_{20} \cdot \text{LDF} + \\ & A_{21} \cdot \text{SDF} + A_{22} \cdot \text{NDP} + A_{23} \cdot \text{SMD} + A_{24} \cdot \text{LFS} + E_1 \quad (1) \end{aligned}$$

$E_1$  is the error in the model and the  $A_i$ 's were to be determined by the multiple regression procedure.

Model 2 (the Cromer digit model) was constructed using the 4 Cromer digit variables (TDF, UDF, TDS, and UDS) to see how it compared with Model 1.

$$\text{DIFF} = A_0 + A_2 \cdot \text{TDF} + A_3 \cdot \text{UDF} + A_4 \cdot \text{TDS} + A_5 \cdot \text{UDS} + E_2 \quad (2)$$

where  $E_2$  is the error in the model.

Model 3 (the Cromer process model) used the 9 process variables (OA, OM, DCA, DCM, LDF, SDF, NDP, SMD, and LFS) and was also compared with Model 1.

$$\begin{aligned} \text{DIFF} = & A_0 + A_{16} \cdot \text{OA} + A_{17} \cdot \text{OM} + A_{18} \cdot \text{DCA} + A_{19} \cdot \text{DCM} + \\ & A_{20} \cdot \text{LDF} + A_{21} \cdot \text{SDF} + A_{22} \cdot \text{NDP} + A_{23} \cdot \text{SMD} + A_{24} \cdot \\ & \text{LFS} + E_3 \quad (3) \end{aligned}$$

Model 4 (the flow chart complete model) was constructed using the 10 new variables along with the 14 Cromer variables except OA and OM. This model was then compared with Model 1 to see if it accounted for more of the variance in the difficulty level.

$$\begin{aligned} \text{DIFF} = & A_0 + A_1 \cdot \text{ORD} + A_2 \cdot \text{TDF} + A_3 \cdot \text{UDF} + A_4 \cdot \text{TDS} + A_5 \cdot \\ & \text{UDS} + A_6 \cdot \text{NDM} + A_7 \cdot \text{YDM} + A_8 \cdot \text{NOM} + A_9 \cdot \text{NWM} + A_{10} \cdot \\ & \text{TSM} + A_{11} \cdot \text{NDA} + A_{12} \cdot \text{YDA} + A_{13} \cdot \text{NOA} + A_{14} \cdot \text{NWA} + \\ & A_{15} \cdot \text{TSA} + A_{18} \cdot \text{DCS} + A_{19} \cdot \text{DCM} + A_{20} \cdot \text{LDF} + A_{21} \cdot \\ & \text{SDF} + A_{22} \cdot \text{NDP} + A_{23} \cdot \text{SMD} + A_{24} \cdot \text{LFS} + E_4 \quad (4) \end{aligned}$$

Model 5 (the flow chart--process model) used the 10 new variables with the 9 Cromer process variables except OA and OM. This model was compared with Models 1, 3, and 4.

$$\begin{aligned} \text{DIFF} = & A_0 + A_6 \cdot NDM + A_7 \cdot YDM + A_8 \cdot NOM + A_9 \cdot NWM + A_{10} \cdot \\ & TSM + A_{11} \cdot NDA + A_{12} \cdot YDA + A_{13} \cdot NOA + A_{14} \cdot NWA + \\ & A_{15} \cdot TSA + A_{18} \cdot DCA + A_{19} \cdot DCM + A_{20} \cdot LDF + A_{21} \cdot \\ & SDF + A_{22} \cdot NDP + A_{23} \cdot SMD + A_{24} \cdot LFS + E_5 \end{aligned} \quad (5)$$

Model 6 (the flow chart only model) was designed to see how much of the variance was accounted for by the 10 new factors. It was then compared with Models 1, 3, 4, and 5.

$$\begin{aligned} \text{DIFF} = & A_0 + A_6 \cdot NDM + A_7 \cdot YDM + A_8 \cdot NOM + A_9 \cdot NWM + A_{10} \cdot \\ & TSM + A_{11} \cdot NDA + A_{12} \cdot YDA + A_{13} \cdot NOA + A_{14} \cdot NWA + A_{15} \cdot \\ & TSA + E_6 \end{aligned} \quad (6)$$

Model 7 (the flow chart--digit model) used the 10 new variables along with Cromer's 4 digit variables. It was compared with Models 1, 4, and 6.

$$\begin{aligned} \text{DIFF} = & A_0 + A_2 \cdot TDF + A_3 \cdot UDF + A_4 \cdot TDS + A_5 \cdot UDS + A_6 \cdot \\ & NDM + A_7 \cdot YDM + A_8 \cdot NOM + A_9 \cdot NWM + A_{10} \cdot TSM + A_{11} \cdot \\ & NDA + A_{12} \cdot YDA + A_{13} \cdot NOA + A_{14} \cdot NWA + A_{15} \cdot TSA + E_7 \end{aligned} \quad (7)$$

### Reduced Models

Since the number of steps of each type involved in both the multiplication and addition routines is partly determined by the number of digits carried in multiplication (DCM) and addition (DCA), it was decided to consider 2 models that deleted the two variables DCM and DCA.

Model 8 (reduced Model 5) was constructed using the 10 new variables and the Cromer process variables except OA, OM, DCA, and DCM. This model was compared with Models 3, 5, and 6.

$$\begin{aligned} \text{DIFF} = & A_0 + A_6 \cdot NDM + A_7 \cdot YDM + A_8 \cdot NOM + A_9 \cdot NWM + \\ & A_{10} \cdot TSM + A_{11} \cdot NDA + A_{12} \cdot YDA + A_{13} \cdot NOA + \\ & A_{14} \cdot NWA + A_{15} \cdot TSA + A_{20} \cdot LDF + A_{21} \cdot SDF + A_{22} \cdot \\ & NDP + A_{23} \cdot SMD + A_{24} \cdot LFS + E_8 \end{aligned} \quad (8)$$

Model 9 (reduced Model 4) used the 10 new variables, the 4 Cromer digit variables and the Cromer process variables except OA, OM, DCA, and DCM. This model was compared with Models 1, 4, 7, and 8.

$$\begin{aligned}
 \text{DIFF} = & A_0 + A_2 \cdot \text{TDF} + A_3 \cdot \text{UDF} + A_4 \cdot \text{TDS} + A_5 \cdot \text{UDS} + A_6 \cdot \\
 & \text{NDM} + A_7 \cdot \text{YDM} + A_8 \cdot \text{NOM} + A_9 \cdot \text{NWM} + A_{10} \cdot \text{TSM} + \\
 & A_{11} \cdot \text{NDA} + A_{12} \cdot \text{YDA} + A_{13} \cdot \text{NOA} + A_{14} \cdot \text{NWA} + A_{15} \cdot \\
 & \text{TSA} + A_{20} \cdot \text{LDF} + A_{21} \cdot \text{SDF} + A_{22} \cdot \text{NDP} + A_{23} \cdot \text{SMD} + \\
 & A_{24} \cdot \text{LFS} + E_9
 \end{aligned} \tag{9}$$

Also, Cromer included the variable SMD, which equals 1 if the tens digit and the units digit of either of the factors are equal. Otherwise, the value of SMD equals 0. It would be expected that this factor would correlate negatively with the difficulty (in Cromer's study it does, in fact). However, the only case in which the digits' being the same would be an important consideration is when the units and the tens digits of the second factor are the same, thus allowing the student to only actually do one multiplication. In the case where the units and tens digits of the first factor are the same, it would not affect the problem so much. Since the multiplication routine considers this possibility, it was decided to construct two additional models deleting SMD.

Model 10 (reduced process model) was constructed using the 10 new variables and the Cromer process variables LDF, SDF, NDP, and LFS. This model was compared with Models 3, 5, 6, and 8.

$$\begin{aligned}
 \text{DIFF} = & A_0 + A_6 \cdot \text{NDM} + A_7 \cdot \text{YDM} + A_8 \cdot \text{NOM} + A_9 \cdot \text{NWM} + \\
 & A_{10} \cdot \text{TSM} + A_{11} \cdot \text{NDA} + A_{12} \cdot \text{YDA} + A_{13} \cdot \text{NOA} + A_{14} \cdot \\
 & \text{NWA} + A_{15} \cdot \text{TSA} + A_{20} \cdot \text{LDF} + A_{21} \cdot \text{SDF} + A_{22} \cdot \text{NDP} + \\
 & A_{24} \cdot \text{LFS} + E_{10}
 \end{aligned} \tag{10}$$

Model 11 (reduced complete model) used the 10 new variables Cromer's digit variables, and the Cromer process variables LDF, SDF, NDP, and LFS. This model was then compared with Models 1, 4, 7, 9, and 10.

$$\begin{aligned}
 \text{DIFF} = & A_0 + A_2 \cdot \text{TDF} + A_3 \cdot \text{UDF} + A_4 \cdot \text{TDS} + A_5 \cdot \text{UDS} + A_6 \cdot \\
 & \text{NDM} + A_7 \cdot \text{YDM} + A_8 \cdot \text{NOM} + A_9 \cdot \text{NWM} + A_{10} \cdot \text{TSM} + \\
 & A_{11} \cdot \text{NDA} + A_{12} \cdot \text{YDA} + A_{13} \cdot \text{NOA} + A_{14} \cdot \text{NWA} + A_{15} \cdot \\
 & \text{TSA} + A_{20} \cdot \text{LDF} + A_{21} \cdot \text{SDF} + A_{22} \cdot \text{NDP} + A_{24} \cdot \text{LFS} + \\
 & E_{11}
 \end{aligned} \tag{11}$$

### Factor Analytic Models

Since there is considerable correlation between the 24 variables, it was decided to perform a factor analysis, as Cromer had done, of the variables used in all 11 of the models except Model 2. Cromer had used a principle axis factor analysis with promax rotations. The promax rotations yielded oblique factors. These procedures were not available on the Univac 1110 at the University of Wisconsin-Madison, and so a principle factor analysis with varimax rotations was used. Since NDM, NOM, NWM, and TSM are linearly dependent, as are NDA, NOA, NWA, and TSA, and since the principle factor analysis method available could not handle such a situation, TSM and TSA were removed before the factor analysis was performed. The varimax rotations yielded orthogonal factors, and so the results are not comparable with those of Cromer, and in some cases different factors are extracted. The digit variables in Model 2 were not factored, since the numbers were chosen randomly and, therefore, four orthogonal factors could be expected.

### One-Variable Models

Since there were 24 predictor variables (Cromer's 14 together with 10 new ones), 24 models consisting of one variable each were considered in order to test the amount of the variation that each variable accounted for.

$$\text{DIFF} = A_0 + A_i \cdot w_i + E_{13+i}$$

where  $w_i$  = the  $i$ th of the 24 possible predictor variables

$E_{13+i}$  = the error of the model

### Restricted Models

Next, for each of Models 1-11, systematic restricted models were considered by eliminating one variable from consideration at a time and then replacing that variable and removing the next. This process was continued until all variables had been removed once. This was done to test the independent contribution of each variable.

After the factors were extracted, the variable correlating most highly with that factor was used to construct a new model. Models 1F and 3F-11F, parallel to Models 1 and 3-11, were then constructed and compared with the fast models to see if they accounted for approximately the same amount of the variance. Only factors accounting for more than 1 percent of the total factor variance were included in the analysis. The factors which Cromer extracted from Models 1 and 3 were different from the factors obtained in this study, and therefore two additional models, 1FC and 3FC, were constructed using the factors that he had obtained.

## II

### RESULTS

In first analyzing the variables for each of the 168 multiplication problems, a correlation matrix was constructed. This matrix was used, with the twenty-fifth row deleted and certain other rows and columns deleted, to perform the factor analysis. The correlation matrix is found in Table 1. All of the variables except ORD and SMD correlated significantly with DIFF. The highest correlation with DIFF was TSM, which accounted for 53 percent of the variance, and the lowest after ORD and SMD was LFS, which accounted for 3 percent. The 10 new variables accounted for a significant part of the variance, with the lowest being YDM and YDA, which accounted for 7.8 percent and 13.2 percent of the variance, respectively.

In this section, the data for the 11 models along with the models derived from the factor analysis are reported. Also, comparisons are made with similar models to note any differences. Data for the one-variable models are summarized.

Model 1. This complete Cromer model produced an  $R^2$  value of .7763 and a corrected  $R^2$  of .7558. The data for Model 1 are presented in Table 2.

When the factor analysis was performed on the variables in Model 1 (13 process and digit variables), 8 factors were extracted. The eighth factor was not considered, since it only accounted for .1 percent of the total factor variance. Table 3 indicates the percentage of the total factor variance and the percentage of the total variance accounted for.

Table 4 presents the rotated factor matrix. Using this table, the variables that correlated most highly with each factor were chosen to represent that factor in the factor model. The variables used were DCM, TDS, DCA, TDF, UDS, LDF, and SMD. The Model 1F generated by the use of these 7 factors is presented in Table 5. This model yielded an  $R^2$  of .7053 and a corrected  $R^2$  of .6924. Since Cromer's factoring procedures yielded oblique factors, the factors that he extracted were somewhat different. Using only 4 factors and the variables DCM, TDS, OA, and TDF to represent them, the model yielded an  $R^2$  of .6572 and a corrected  $R^2$  of .6487. The summary of Model 1FC is presented in Table 6.

In both cases, the models generated from the factors extracted by the factor analysis accounted for less of the variance than did the full model. A summary of the differences in the  $R^2$  for the three models appears in Table 7.

Summaries of the restricted models produced from each of the preceding models are presented in Tables 59, 60, and 61 in Appendix B.



TABLE 2  
BASIC REGRESSION STATISTICS  
MODEL 1

Variable	Type	Variables in the Equation						Sig. Level
		Regression Coefficient	Std. Error of Regression Coefficient	Standardized Regression Coefficient	Partial Correlation Coefficient	T-Value with 153 Deg. Freedom	Partial F Value with 1 and 153 Deg. Freedom	
25 DIFF	dependent							
	constant	-.1258684	.0379422			-2.259	-3.31737	11.00498
1 ORD	independent	.0001001	.0001699	.02333	.048	.58898	.34690	.5567
2 TDF	independent	.0028766	.0026061	.0726	.089	1.10380	1.21837	.2714
3 UDF	independent	.0094763	.0024531	.2675	.298	3.86304	14.92304	.0002
4 TDS	independent	.0082645	.0029967	.2326	.218	2.75782	7.60558	.0065
5 UDS	independent	.0069716	.0022338	.1871	.245	3.12099	9.74060	.0022
16 OA	independent	.0275044	.0077889	.3724	.275	3.53123	12.46962	.0005
17 OM	independent	-.0143742	.0082202	-.2864	-.140	-1.74864	3.05772	.0824
18 DCA	independent	-.0221403	.0125634	-.1546	-.141	-1.76229	3.10566	.0800
19 DCM	independent	.0347200	.0183326	.2768	.151	1.89493	3.59077	.0600
20 LDF	independent	.0068338	.0041703	.0966	.131	1.63869	2.68529	.1033
21 SDF	independent	.0078841	.0051477	.1242	.123	1.53157	2.34572	.1277
22 NDP	independent	.0331459	.0118151	.2071	.221	2.80539	7.87023	.0057
23 SMD	independent	-.0434748	.0108707	-.1623	-.308	-3.99926	15.99410	.0001
24 LFS	independent	-.0013654	.0146420	-.0064	-.008	-.09326	.00870	.9258

ANALYSIS OF VARIANCE SUMMARY TABLE

Source of Variation	Sum of Squares	Deg. Freedom	Mean Square
Linear Regression	1.40818		
Residuals from Regression	.40589		
Corrected Total	1.81407		
F-Ratio = 37.92 with 14 and 153 Deg. Freedom			
Significance Level of F-Ratio = .0000			
Correction for Mean	6.74803	1	
Uncorrected Total	8.56210	168	

TABLE 3  
PROPORTION OF VARIANCE ACCOUNTED  
FOR BY THE FACTORS  
MODEL 1

Factor Number	Percentage of Total Factor Variance	Percentage of Total Variance
1	28.0	20.9
2	21.0	15.7
3	17.5	13.1
4	13.1	9.8
5	8.6	6.4
6	7.7	5.7
7	4.1	3.1







TABLE 7  
COMPARISON OF  $R^2$  FOR MODELS 1, 1F, AND 1C

Full Model	Restricted Model	$R^2$ Diff.	Corr. $R^2$ Diff.
Model 1	Model 1F	.0710	.0634
Model 1	Model 1FC	.1191	.1071
Model 1F	Model 1FC	.0481	.0437

Model 2. The data for this model (using the four digit variables) are presented in Table 8. The model yielded an  $R^2$  of .6879 and a corrected  $R^2$  of .6802. The  $R^2$  difference between Models 1 and 2 is presented in Table 9. A summary of the restricted models generated is presented in Table 62 in Appendix B.

Model 3. Cromer's process model (nine process variables) yielded an  $R^2$  of .7454 and a corrected  $R^2$  of .7309. A summary of the data for the model is presented in Table 10. The model thus accounted for less of the variance than did Model 1, but for more than Model 2. The factor analysis of these nine variables yielded five factors of which only four, accounting for 98.5 percent of the total factor variance, were considered. The percentage of the total factor variance and the percentage of the total variance accounted for are presented in Table 11. Table 12 presents the rotated factor matrix, which was used to see which variable correlated most highly with each factor. The variables DCM, OA, NDP, and SMD were then used to construct Model 3F. This factor model yielded an  $R^2$  of .6628 and a corrected  $R^2$  of .6545 and is presented in Table 13.

As with Model 1, the factoring procedure used by Cromer produced different factors from those in Model 3F. The factors he used were OM, OA, and NDP. These factors were used to produce Model 3FC, which is presented in Table 14. This model gave an  $R^2$  of .6049 and a corrected  $R^2$  of .5977.

In both cases, the factor models accounted for less of the variance than did the full model. The factors extracted in this study accounted for significantly more of the variance than did Cromer's factors. A summary of these comparisons is presented in Table 15. The restricted models generated from these three models are summarized in Tables 63, 64, and 65 in Appendix B.

Model 4. This complete flow chart model replaced the 2 Cromer variables OA and OM with the 10 new factors. In this model and in all the other complete models, TSM and TSA were not included, due to the linear dependence of the variables when the variables were included. Model 4 yielded an  $R^2$  of .7855 and a corrected  $R^2$  of .7564. These data for Model 4 are presented in Table 16. Model 4 accounted for somewhat more of the variance than did Model 1 for both the  $R^2$  and corrected  $R^2$  values.

The factor analysis of the variables in this model, omitting TSM and TSA, extracted 12 factors. Of these 12, 3 accounted for less than 1 percent of the total factor variance and were therefore not considered further. The percentage of total factor variance and the percentage of total variance for each of the remaining 9 factors are presented in Table 17. The rotated factor matrix for the 9 factors is presented in Table 18. The variables correlating most highly with each factor were chosen to represent that factor in the factor model. The variables that were used were NWA, YDM, YDA, LFS, UDS, LDF, SMD, NDP, and SDF. The model generated from these 9 variables, Model 4F, yielded an  $R^2$  of .8521 and a corrected  $R^2$  of .7105 (see Table 19). A summary of the comparison between Models 1, 1F, 4, and 4F is presented in Table 20. A summary of the restricted models for Model 4 is presented in Table 66 in Appendix B.

TABLE 8  
BASIC REGRESSION STATISTICS  
MODEL 2

Variables in the Equation					
Variable	Type	Regression Coefficient	Std. Error of Regression Coefficient	Partial Correlation Coefficient	T-Value with 163 Deg. Freedom
25 DIFF	dependent				
2 constant		-.062142	.015824	-.294	-3.92698
2 TDF	Independent	.008252	.001749	.347	4.71828
3 UDF	Independent	.016824	.001553	.647	10.83299
4 TDS	Independent	.017945	.001565	.668	11.46772
5 UDS	Independent	.014583	.001633	.573	8.92913

## ANALYSIS OF VARIANCE SUMMARY TABLE

Source of Variation	Sum of Squares	Deg. Freedom	Mean Square
Linear Regression	1.24791	4	.31198
Residuals from Regression	.56617	163	.00347
Corrected Total	1.81407	167	
F-Ratio = 89.82 with 4 and 163 Deg. Freedom			
Significance Level of F-Ratio = .0000			
Correction for Mean	6.74803	1	
Uncorrected Total	8.56210	168	

TABLE 9  
COMPARISON OF  $R^2$  FOR MODELS 1 AND 2

Full Model	Restricted Model	$R^2$ Diff.	Corr. $R^2$ Diff.
Model 1	Model 2	.0884	.0756



TABLE 11  
PROPORTION OF VARIANCE ACCOUNTED  
FOR BY THE FACTORS  
MODEL 3

Factor Number	Percentage of Total Factor Variance	Percentage of Total Variance
1	42.4	26.0
2	41.1	25.2
3	8.2	5.0
4	6.9	4.2

TABLE 12  
ROTATED FACTOR MATRIX  
MODEL 3

Var./Factor	1	2	3	4
1 OA	.370	.831	-.001	.277
2 OM	.864	.393	.229	.029
3 DCA	.240	.817	-.166	.297
4 DCM	.909	.243	.166	.030
5 LDF	.155	.462	.125	-.083
6 SDF	.664	.421	.217	.238
7 NDP	.309	.551	.386	-.097
8 SMD	.028	.043	.070	.365
9 LFS	.109	.006	.359	.093

TABLE 13  
BASIC REGRESSION STATISTICS  
MODEL 3F

		Variables in the Equation					
Variable	Type	Regression Coefficient	Std. Error of Regression Coefficient	Standardized Regression Coefficient	Partial Correlation Coefficient	T-Value with 163 Deg. Freedom	Partial F Value with 1 and 163 Deg. Freedom
25 DIFF	dependent						
	constant	-.05026	.02867		-.136	-1.75310	3.07334
19 DCM	independent	.04437	.00692	.35337	.449	6.40853	.0815
16 OA	independent	.02616	.00447	.3542	.417	5.85527	.0000
22 NDP	independent	.04465	.00891	.2790	.365	5.01103	.0000
23 SMD	independent	-.04313	.01238	-.1610	-.263	-3.48363	.0006

ANALYSIS OF VARIANCE SUMMARY TABLE					
Source of Variation	Sum of Squares	Deg. Freedom	Mean Square		
Linear Regression	1.20231	4	.30058		
Residuals from Regression	.61176	163	.00375		
Corrected Total	1.81407	167			
F-Ratio = 80.09 With 4 and 163 Deg. Freedom					
Significance Level of F-Ratio = .0000					
Correction for Mean	6.74803	1			
Uncorrected Total	8.56210	168			

TABLE 14  
BASIC REGRESSION STATISTICS  
MODEL 3FC

Variables in the Equation						
Variable	Type	Regression Coefficient	Std. Error of Regression Coefficient	Standardized Regression Coefficient	Partial Correlation Coefficient	T-Value with 164 Deg. Freedom
25 DIFF	dependent	-.04658	.03102	-.3386	-.116	-1.50192
17 GM	independent	.01699	.00357	.3386	.349	4.76605
16 OA	independent	.02361	.00497	.3197	.348	4.75291
22 NDP	independent	.03894	.01025	.2434	.284	3.79980

ANALYSIS OF VARIANCE SUMMARY TABLE				
Source of Variation	Sum of Squares	Deg. Freedom	Mean Square	
Linear Regression	1.09731	3	.36577	
Residuals from Regression	.71676	164	.00437	
Corrected Total	1.81407	167		
F-Ratio = 83.69 with 3 and 164 Deg. Freedom				
Significance Level of F-Ratio = .0000				
Correction for Mean	6.74803	1		
Uncorrected Total	8.56210	168		

TABLE 15  
COMPARISON OF  $R^2$  FOR MODELS 1, 3, 3F, AND 3FC

Full Model	Restricted Model	$R^2$ Diff.	Corr. $R^2$ Diff.
Model 1	Model 3	.0309	.0249
Model 3	Model 3F	.0826	.0764
Model 3	Model 3FC	.1449	.1332
Model 3F	Model 3FC	.0579	.0568



**TABLE 17**  
**PROPORTION OF VARIANCE ACCOUNTED**  
**FOR BY THE FACTORS**  
**MODEL 4**

Factor Number	Percentage of Total Factor Variance	Percentage of Total Variance
1	36.6	31.3
2	16.0	13.7
3	14.1	12.0
4	8.5	7.3
5	8.2	7.0
6	6.2	5.3
7	3.9	3.4
8	2.6	2.2
9	2.2	1.9

**TABLE 18**  
**ROTATED FACTOR MATRIX**  
**MODEL 4**

<b>Var./Factor</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
1 TDF	.201	-.088	.068	-.753	-.098	.300	-.099	.106	.124
2 UDF	-.037	.772	.118	-.022	-.027	.240	-.108	-.185	.041
3 TDS	.685	.018	-.029	.308	-.241	.352	.138	.190	.241
4 UDS	.270	.003	.159	.055	.806	.100	.141	-.012	.004
5 NDM	.814	.076	.138	.050	.420	-.068	-.144	.120	.029
6 YDM	-.071	.886	-.009	.061	-.127	-.048	.026	.202	-.106
7 NOM	.656	.542	.123	.045	.361	.027	-.212	-.042	.189
8 NWM	.890	.102	.202	.107	.217	.067	-.212	.022	.063
9 NDA	.866	-.012	.330	.026	.143	.156	.103	-.202	-.125
10 YDA	.222	.049	.885	.045	.010	.074	.101	-.069	-.173
11 NOA	.649	.111	.637	-.097	.237	.150	.132	-.047	.158
12 NWA	.904	-.017	.268	.004	.054	.166	.114	-.130	-.047
13 DCA	.268	.145	.837	-.168	.211	.147	.130	.140	.251
14 DCM	.455	.801	.091	.048	.277	.037	.068	-.021	.111
15 LDF	.223	.153	.178	-.100	.108	.749	-.026	.009	.004
16 SDF	.607	.445	.171	-.074	.174	.091	.268	-.084	.282
17 NDP	.822	.061	.112	-.088	.018	.195	.013	.429	.011
18 SMD	-.018	-.017	.107	.034	.060	-.009	.548	.003	.007
19 LFS	.263	-.011	-.025	.798	-.015	.109	.142	.074	.090

TABLE 19  
BASIC REGRESSION STATISTICS  
MODEL 4F

Variables in the Equation								
	Variable	Type	Regression Coefficient	Std. Error of Regression Coefficient	Standardized Regression Coefficient	Partial Correlation Coefficient	T-Value with 158 Deg. Freedom	Partial F Value with 1 and 158 Deg. Freedom
25	DIFF	dependent	-16.2541	3.3238			-4.89023	23.91435
14	NWA	independent	1.1477	.5437	.1823	.166	2.11091	<b>4.45595</b>
7	YDM	independent	2.7755	.7397	.1926	.286	3.75205	.0002
12	YDA	independent	.7375	.5960	.0614	.098	1.23749	.2177
24	LFS	independent	.6383	.9177	.0301	.055	.69554	.4877
5	UDS	independent	.3980	.1778	.1074	.175	2.23882	.0266
20	LDF	independent	1.9599	.3293	.2771	.428	5.95112	.0000
23	SMD	independent	-4.9882	1.1580	-1.862	-.324	-4.30776	18.55677
22	NDP	independent	2.3050	.9718	.1542	.185	2.37200	5.62637
21	SDF	independent	1.9972	.4171	.3145	.356	4.78806	.0189

ANALYSIS OF VARIANCE SUMMARY TABLE				
Source of Variation	Sum of Squares	Deg. Freedom	Mean Square	
Linear Regression	13171.82620	9	1463.53624	
Residuals from Regression	4968.88213	158	31.44862	
Corrected Total	18140.70833	167		
F-Ratio = 46.54 with 9 and 158 Deg. Freedom				
Significance Level of F-Ratio = .0000				
Correction for Mean	67480.29167	1		
Uncorrected Total	85621.00000	168		

TABLE 20  
COMPARISON OF  $R^2$  FOR MODELS 1, 1F, 4, AND 4F

Full Model	Restricted Model	$R^2$ Diff.	Corr. $R^2$ Diff.
Model 1	Model 4	-.0116	.0001
Model 1	Model 4F	-.0758	.0453
Model 4	Model 4F	-.0666	.0006

Model 5. This model replaced the variables OA and OM in the set of Cromer process variables with the 10 new variables. Model 5 yielded an  $R^2$  of .7405 and a corrected  $R^2$  of .7149. The data for this model are presented in Table 21. This model accounted for less of the variance than did Model 4 and accounted for somewhat less than did Model 3.

The variables, omitting TSM and TSA, were then subjected to a factor analysis which yielded 10 factors. Of these factors, 2 were dropped from consideration since combined they accounted for .4 percent of the total factor variance. Table 22 presents the percentage of total factor variance and the percentage of total variance accounted for by the remaining 8 variables.

Table 23 presents the rotated factor matrix. This matrix was used to choose the variable that correlated most highly with each variable. These variables, NDM, YDA, YDM, LDF, SMD, LFS, NDA, and NDP, were then used to construct Model 5F. The data for the model are presented in Table 24. Model 5F yielded an  $R^2$  of .6926 and a corrected  $R^2$  of .6771, which accounted for less of the variation than did the full model, but which accounted for more than the Cromer process factor models (3F and 3FC). These comparisons are summarized in Table 25. A summary of the restricted models produced from these models is presented in Tables 67 and 68 in Appendix B.

Model 6. This model was produced using only the 10 new variables and is presented in Table 26. This model yielded an  $R^2$  of .6400 and a corrected  $R^2$  of .6219. Model 6 accounted for less of the variation than did Models 3 and 5.

The 8 variables, omitting TSM and TSA, were subjected to a factor analysis which yielded 4 factors. Table 27 gives the percentage of total factor variance and the percentage of total variance accounted for by each of these 4 factors.

Table 28 is the rotated factor matrix which was used to determine the variables that corresponded most highly with each factor. These variables, NDM, YDA, NWA, and YDM, were then used to construct the factor model. Model 6F yielded an  $R^2$  of .5877 and a corrected  $R^2$  of .5775. The data are presented in Table 29. Again, this factor model accounted for less of the variance than did the full model (Model 6) and also for less than the other process models (Models 3 and 5). Table 30 summarizes these comparisons. Tables 69 and 70 in Appendix B are a summary of the restricted models generated from these two models.

Model 7. This model was constructed using the 4 Cromer digit variables and the 10 new variables. Data for Model 7 are presented in Table 31. It yielded an  $R^2$  of .7613 and a corrected  $R^2$  of .7425. Compared with Models 1 and 4, this model accounted for less of the variance, but the difference was fairly small. The 12 variables in this model, omitting TSM and TSA, were subjected to a factor analysis which yielded 7 factors. Table 32 presents the percentage of total factor variance and the percentage of total variance accounted for by each factor. Using the rotated factor matrix found in Table 33, the variable correlating most highly with each factor was chosen to represent that factor in the factor model. The variables chosen to



**TABLE 22**  
**PROPORTION OF VARIANCE ACCOUNTED  
 FOR BY THE FACTORS**  
**MODEL 5**

Factor Number	Percentage of Total Factor Variance	Percentage of Total Variance
1	43.5	35.2
2	18.5	15.0
3	15.8	12.8
4	7.2	5.8
5	4.7	3.8
6	4.1	3.3
7	2.9	2.4
8	2.9	2.3

TABLE 23  
ROTATED FACTOR MATRIX  
MODEL 5

Var./Factor		1	2	3	4	5	6	7	8
1	NDM	.910	.169	.087	-.038	-.104	.165	-.127	.037
2	YDM	-.102	.008	.905	-.003	-.018	.022	.012	.128
3	NOM	.740	.143	.513	.153	-.181	.123	-.127	-.232
4	NWM	.884	.221	.089	.160	-.179	.289	-.033	.015
5	NDA	.789	.342	-.058	.281	.111	.165	.340	-.063
6	YDA	.162	.896	.030	.090	.091	.096	.163	.024
7	NOA	.668	.625	.062	.305	.192	-.048	-.005	-.089
8	NWA	.802	.264	-.062	.329	.143	.145	.339	.019
9	DCA	.329	.814	.112	.277	.190	-.181	-.251	.001
10	DCM	.499	.107	.786	.189	.081	.087	-.037	-.213
11	LDF	.200	.213	.090	.492	-.033	.030	.017	.020
12	SDF	.621	.173	.397	.239	.298	.002	-.027	-.149
13	NDP	.743	.108	.091	.348	.018	.088	-.031	.445
14	SMD	-.040	.111	.004	-.016	.525	.090	.009	.003
15	LFS	.182	-.019	.046	.023	.105	.502	.014	.006

TABLE 24  
BASIC REGRESSION STATISTICS  
MODEL 5F

		Variables in the Equation									
Variable	Type	Regression Coefficient	Std. Error of Regression Coefficient	Standardized Regression Coefficient	Partial Correlation Coefficient	T-Value with 159 Deg. Freedom	Partial F Value with 1 and 159 Deg. Freedom	Sig. Level			
25 DIFF	dependent										
	constant	-.268449	.037628	.2713	-.492	-7.13433	50.89865	.0000			
6 NDM	Independent	.020556	.005853	.0193	.268	3.51190	12.33346	.0006			
12 YDA	Independent	.002307	.006596	.0193	.028	.34979	.12235	.7270			
7 YDM	Independent	.042467	.006735	.2935	.447	6.30563	39.76093	.0000			
20 LDF	Independent	.024109	.003532	.3409	.476	6.82589	46.59274	.0000			
23 SMD	Independent	-.027921	.012169	-.1042	-.179	-2.29435	5.26402	.0231			
24 LFS	Independent	.001206	.009704	.0057	.010	.12428	.01544	.9013			
11 NDA	Independent	.018612	.005393	.3008	.264	3.45098	11.90929	.0007			
22 NDP	Independent	.010669	.011567	.0667	.073	.92238	.85079	.3577			

ANALYSIS OF VARIANCE SUMMARY TABLE

Source of Variation	Sum of Squares	Deg. Freedom	Mean Square
Linear Regression	1.25641	8	
Residuals from Regression	.55766	159	.00351
Corrected Total	1.81407	167	
F-Ratio = 44.78 with 8 and 159 Deg. Freedom			
Significance Level of F-Ratio = .0000			
Correction for Mean	6.74803	1	
Uncorrected Total	8.56210	168	

TABLE 25  
COMPARISON OF  $R^2$  FOR MODELS 3, 3F, 3FC, 4, 5, AND 5F

Full Model	Restricted Model	$R^2$ Diff.	Corr. $R^2$ Diff.
Model 3	Model 5	.0049	.0160
Model 4	Model 5	.0450	.0415
Model 5	Model 5F	.0479	.0378
Model 3F	Model 5F	-.0300	-.0226
Model 3FC	Model 5F	-.0857	-.0949

TABLE 26  
BASIC REGRESSION STATISTICS  
MODEL 6

		Variables in the Equation									
Variable	Type	Regression Coefficient	Std. Error of Regression Coefficient	Standardized Regression Coefficient	Partial Correlation Coefficient	T-Value with 159 Deg. Freedom	T-Value with 1 and 159 Deg. Freedom	Partial F Value with 1 and 159 Deg. Freedom	Sign. Level		
25 DIFF	dependent	- .028092	.037017	-	-	-	-	-			
6 NDM	Independent	- .007109	.009797	- .0938	- .057	- .72560	.52650	.4490	.4692		
7 YDM	Independent	.033855	.009371	.2340	.275	3.61285	13.05267		.0004		
8 NOM	Independent	.008012	.004741	.1980	.133	1.68988	2.85571		.0930		
9 NNM	Independent	.029702	.020176	.2494	.116	1.47216	2.16724		.1430		
11 NDA	Independent	- .007123	.014551	- .1151	-.039	-.48954	.23965		.6251		
12 YDA	Independent	- .009197	.009488	-.0769	-.077	-.96927	.93948		.3339		
13 NOA	Independent	.016229	.005408	.3583	.232	3.00070	9.00421		.0031		
14 NWA	Independent	.017351	.013915	.2760	.098	1.24690	1.55477		.2143		

## ANALYSIS OF VARIANCE SUMMARY TABLE

Source of Variation	Sum of Squares	Deg. Freedom	Mean Square
Linear Regression	1.16109	8	.14514
Residuals from Regression	.65298	159	.00411
Corrected Total	1.81407	167	
F-Ratio = 35.34 with 8 and 159 Deg. Freedom			
Significance Level of F-Ratio = .0000			
Correction for Mean	6.74803	1	
Uncorrected Total	8.56210	168	

TABLE 27  
PROPORTION OF VARIANCE ACCOUNTED  
FOR BY THE FACTORS\*  
MODEL 6

Factor Number	Percentage of Total Factor Variance	Percentage of Total Variance
1	49.1	41.1
2	26.3	22.0
3	13.6	11.4
4	11.0	9.2

TABLE 28  
ROTATED FACTOR MATRIX  
MODEL 6

Var./Factor	1	2	3	4
1 NDM	.911	.202	.137	-.015
2 YDM	.028	-.007	-.064	.695
3 NOM	.786	.179	.191	.453
4 NWM	.896	.271	.263	.011
5 NDA	.612	.510	.557	-.157
6 YDA	.122	.798	.108	.030
7 NOA	.538	.722	.269	.006
8 NWA	.598	.442	.623	-.156

2\*



TABLE 30  
COMPARISON OF  $R^2$  FOR MODELS 3, 3F, 3FC, 5, 5F, 6, AND 6F

Full Model	Restricted Model	$R^2$	Corr. $R^2$
		Diff.	Diff.
Model 3	Model 6	.1054	.1090
Model 5	Model 6	.1005	.0930
Model 6	Model 6F	.0523	.0444
Model 3F	Model 6F	.0749	.0770
Model 3FC	Model 6F	.0172	.0202
Model 5F	Model 6F	.0049	.0996



TABLE 32  
PROPORTION OF VARIANCE ACCOUNTED  
FOR BY THE FACTORS  
MODEL 7

Factor Number	Percentage of Total Factor Variance	Percentage of Total Variance
1	37.6	29.0
2	20.0	15.5
3	17.1	13.2
4	16.1	12.4
5	5.4	4.2
6	2.0	1.5
7	1.8	1.4

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TABLE 33  
ROTATED FACTOR MATRIX  
MODEL 7

Var./Factor		1	2	3	4	5	6	7
1	TDF	.095	-.020	.066	-.070	.540	.002	.007
2	UDF	.009	.022	.109	.776	-.003	-.050	.083
3	TDS	.767	-.050	.028	.020	.093	-.062	-.081
4	UDS	.111	.768	.176	-.077	-.049	-.049	.046
5	NDM	.663	.621	.134	.008	.033	.264	-.067
6	YDM	-.006	-.065	-.032	.778	-.116	.051	-.094
7	NOM	.564	.560	.081	.499	.063	.168	-.022
8	NWM	.816	.433	.213	.068	.052	.251	-.038
9	NDA	.736	.336	.473	-.052	.136	-.036	.275
10	YDA	.146	.114	.825	.071	.046	.028	.007
11	NOA	.495	.425	.634	.103	.332	-.044	-.011
12	NWA	.803	.258	.394	-.048	.211	-.097	.237

represent the factors were NWM, UDS, YDA, YDM, TDF, NDM, and NDA. The data for Model 7F are presented in Table 34. Model 7F yielded an  $R^2$  of .6495 and a corrected  $R^2$  of .6342. Model 7F accounted for less of the variance than did Model 7. The comparisons between the  $R^2$  for this model and the other models are summarized in Table 35. A summary of the restricted models generated from these models is found in Tables 71 and 72 in Appendix B.

Model 8. This model used the 10 new variables plus the Cromer process variables LDF, SDF, NDP, SMD, and LFS. Data for Model 8 are presented in Table 36. Model 8 had an  $R^2$  value of .7404 and a corrected  $R^2$  of .7184. When compared on the basis of  $R^2$  to the other process models (Models 3, 5, and 6), Model 8 came out very favorably. The 13 linearly independent variables for Model 8 were subjected to a factor analysis which extracted 8 factors. Table 37 reports the percentage of total variance and the percentage of total variance accounted for by each variable.

Table 38, the rotated factor matrix, was used to determine which of the variables correlated most highly with each of the factors. These variables, NDM, YDA, YDM, LDF, SMD, LFS, NWA, and NDP, were then used to construct Model 8F. Data for Model 8F are found in Table 39. This model had an  $R^2$  of .7016 and a corrected  $R^2$  of .6866. Model 8F accounted for more of the variance than did the factor model of the 10 new variables (Model 6F) and the Cromer process factor model (Model 3F) and for about the same amount as Model 5F. A summary of the differences between this model and the other models is presented in Table 40. The restricted models are summarized in Tables 73 and 74 in Appendix B.

Model 9. This model contains the same 15 variables as the previous model, but has, in addition, the 4 Cromer digit variables. The data are found in Table 41 and had an  $R^2$  of .7831 and a corrected  $R^2$  of .7585. Model 9 was about the same or somewhat better than the process and digit models (1 and 7) when comparing  $R^2$  values and was also better than Model 4 when comparing corrected  $R^2$  values. These 17 variables, omitting TSM and TSA, were then subjected to a factor analysis which identified 10 factors, of which 1 factor was eliminated from consideration, since it accounted for .1 percent of the total factor variance. Table 42 lists the percentage of the total factor variance and the percentage of the total variance accounted for by each variable. Using the rotated factor matrix, found in Table 43, the variable corresponding most highly with each factor was then determined. These variables, NWM, YDM, LFS, YDA, UDS, LDF, SMD, NDA, and SDF, were then used to construct Model 9F. Data for Model 9F are given in Table 44. This model had an  $R^2$  of .7339 and a corrected  $R^2$  of .7187. The comparisons of the  $R^2$  for Models 9 and 9F to the other factor models for the digit and process models are summarized in Table 45. The summary of the restricted models for both Models 9 and 9F is given in Tables 75 and 76 in Appendix B.

Model 10. This model was constructed using the 10 new variables and the Cromer process variables LDF, SDF, NDP, and LFS. Data for Model 10 are presented in Table 46. This model had an  $R^2$  of .7245 and a corrected  $R^2$  of .7032. Model 10 accounted for less of the



TABLE 35  
COMPARISON OF  $R^2$  FOR MODELS 1, 1F, 1FC, 4, 6, 7, AND 7F

Full Model	Restricted Model	$R^2$ Diff.	Corr. $R^2$ Diff.
Model 7	Model 6	-.1213	-.1209
Model 1	Model 7	.0150	.0130
Model 4	Model 7	.0242	.0136
Model 7	Model 7F	.1118	.1086
Model 1F	Model 7F	.0558	.0582
Model 1FC	Model 7F	.0077	.0145



TABLE 37

PROPORTION OF VARIANCE ACCOUNTED  
FOR BY THE FACTORS  
MODEL 8

Factor Number	Percentage of Total Factor Variance	Percentage of Total Variance
1	49.8	36.3
2	14.3	10.5
3	10.1	7.4
4	9.9	7.2
5	5.5	4.0
6	5.1	3.7
7	2.7	2.0
8	2.6	1.9

TABLE 38  
ROTATED FACTOR MATRIX  
MODEL 8

Var./Factor		1	2	3	4	5	6	7	8
1	NDM	.911	.155	-.031	.013	-.107	.209	-.068	.107
2	YDM	.023	-.014	.782	.035	.007	.018	-.021	.008
3	NOM	.805	.118	.412	.174	-.155	.146	.011	-.170
4	NWM	.862	.217	-.018	.212	-.169	.328	.021	.055
5	NDA	.709	.410	-.194	.317	.104	.218	.327	.012
6	YDA	.189	.780	.013	.177	.152	.034	.026	.016
7	NOA	.665	.581	-.026	.342	.198	-.023	.028	-.022
8	NWA	.718	.306	-.194	.369	.138	.199	.361	.093
9	LDF	.193	.189	.062	.505	-.025	.025	.022	.020
10	SDF	.676	.122	.307	.265	.308	.026	.099	-.132
11	NDP	.690	.076	.028	.387	.012	.151	.048	.419
12	SMD	-.042	.099	-.002	-.012	.506	.084	.010	.003
13	LFS	.166	.013	.022	.022	.095	.441	.015	.012

TABLE 39  
BASIC REGRESSION STATISTICS  
MODEL 8F

		Variables in the Equation									
Variable	Type	Regression Coefficient	Std. Error of Regression Coefficient	Standardized Regression Coefficient	Partial Correlation Coefficient	T-Value with 159 Deg. Freedom	P-Value with 1 and 159 Deg. Freedom	Sig. Level			
25 DIFF	dependent										
	constant	-.25008418	.03788006	.3048	-.464	-6.60200	43.58642	.0000			
6 NDN	independent	.02309881	.00533357	.0269	.325	4.33084	18.75613	.0000			
12 YDA	independent	.00321598	.00617629	.041	.52070	.27113	.6033				
7 YDM	independent	.04439289	.00668410	.3068	.64157	44.11039	.0000				
20 LDF	independent	.02447479	.00344288	.3461	.491	7.10882	.50.53530	.0000			
23 SMD	independent	-.02826840	.01197757	-.1055	-.184	-.2.36011	5.57013	.0195			
24 LFS	independent	-.00008086	.00957811	-.0004	-.001	-.00844	.00007	.9933			
14 NWA	independent	.02192096	.00530165	.3406	.312	4.13475	17.09613	.0001			
22 NDP	independent	-.00273262	.01247945	-.0171	-.017	-.21897	.04795	.8270			

ANALYSIS OF VARIANCE SUMMARY TABLE					
Source of Variation	Sum of Squares	Deg. Freedom	Mean Square		
Linear Regression	1.27284	8	.15910		
Residuals from Regression	.54123	159	.00340		
Corrected Total	1.81407	167			
F-Ratio = 46.74 with 8 and 159 Deg. Freedom					
Significance Level of F-Ratio = .0000					
Correction for Mean	6.74803	1			
Uncorrected Total	8.56210	168			

TABLE 40.

**COMPARISON OF  $R^2$  FOR MODELS  
3, 3F, 3FC, 5, 5F, 6, 6F, 8, AND 8F**

Full Model	Restricted Model	$R^2$ Diff.	Corr. $R^2$ Diff.
Model 3	Model 8	.0050	.0125
Model 5	Model 8	.0001	-.0035
Model 6	Model 8	-.1004	-.0865
Model 8	Model 8F	.0523	.0444
Model 3F	Model 8F	-.0390	-.0321
Model 3FC	Model 8F	-.0967	-.0889
Model 5F	Model 8F	-.0090	-.0095
Model 6F	Model 8F	-.1139	-.1091

TABLE 41  
BASIC REGRESSION STATISTICS  
MODEL 9

Variable	Type	Regression Coefficient	Std. Error of Regression Coefficient	Variables in the Equation				Partial F Value with 1 and 150 Deg. Freedom	Sig. Level
				Standardized Regression Coefficient	Partial Correlation Coefficient	T-Value with 150 Deg. Freedom			
25 DIFF	dependent constant	-.182932	.043250			- .328	-4.25273	18.08568	.0000
2 TDF	independent	.006542	.002819	.1651	.186	2.32035	5.38402	.0217	
3 UDF	independent	.011930	.002517	.3368	.361	4.74054	22.4774	.0000	
4 TDS	independent	.009967	.003187	.2806	.247	3.12692	9.77762	.0021	
5 UDS	independent	.010170	.002840	.2729	.281	3.58148	12.82697	.0005	
6 NDM	independent	.010187	.010138	.1344	.082	1.00485	1.00972	.3166	
7 YDM	independent	.023434	.010067	.1620	.187	2.32794	5.41931	.0213	
8 NOM	independent	-.008433	.005147	-.2084	-.133	-1.63855	2.68486	.1034	
9 NWN	independent	.038272	.021278	.3214	.145	1.79863	3.23508	.0741	
11 NDA	independent	-.011905	.014230	-.1924	-.068	-.83664	.69997	.4041	
12 YDA	independent	.014736	.008730	.1232	.137	1.68798	2.84946	.0935	
13 NOA	independent	-.002869	-.005493	-.0633	-.043	-.52225	.27274	.6023	
14 NWA	independent	.015542	.014694	.2472	.086	1.05770	1.11873	.2919	
20 LDF	independent	.006621	.004516	.0936	.119	1.46611	2.14947	.1447	
21 SDF	independent	.007167	.004903	.1129	.119	1.46187	2.13705	.1459	
22 NDP	independent	-.012223	.016157	-.0764	-.062	-.75651	.57230	.4505	
23 SMD	independent	-.040167	.011356	-.1499	-.267	-3.38794	11.47813	.0009	
24 LFS	independent	-.002479	.014655	-.0117	-.014	-.16914	.02861	.8659	

## ANALYSIS OF VARIANCE SUMMARY TABLE

Source of Variation	Sum of Squares	Deg. Freedom	Mean Square
Linear Regression	1.42056	17	.08356
Residuals from Regression	.39351	150	.00262
Corrected Total	1.81402	167	
F-Ratio = 31.85 with 17 and 150 Deg. Freedom			
Significance Level of F-Ratio = .0000			
Correction for Mean	6.74803	1	
Uncorrected Total	8.56210	168	

TABLE 42  
PROPORTION OF VARIANCE ACCOUNTED  
FOR BY THE FACTORS  
MODEL 9

Factor Number	Percentage of Total Factor Variance	Percentage of Total Variance
1	42.9	34.8
2	12.7	10.3
3	9.9	8.0
4	9.8	7.9
5	8.7	7.0
6	7.6	6.2
7	4.5	3.7
8	2.3	1.8
9	1.6	1.3





TABLE 45  
COMPARISON OF  $R^2$  FOR MODELS  
1, 1F, 1FC, 4, 4F, 7, 7F, 8, 9, AND 9F

Full Model	Restricted Model	$R^2$ Diff.	Corr. $R^2$ Diff.
Model 9	Model 8	.0427	.0401
Model 1	Model 9	-.0068	-.0027
Model 4	Model 9	.0024	-.0021
Model 7	Model 9	-.0218	-.0157
Model 9	Model 9F	.0492	.0398
Model 1F	Model 9F	-.0286	-.0243
Model 1FC	Model 9F	-.0767	-.0700
Model 4F	Model 9F	.0516	.0377
Model 7F	Model 9F	.0844	.0845



variance than did the other process models (3, 5, and 8). It accounted for somewhat more of the variance than did Model 6, which consisted of only the 10 new variables.

A factor analysis was then run on these 12 variables, omitting TSM and TSA, to find a smaller set of factors that would account for approximately the same amount of the variance. Eight factors were extracted, of which 1 was eliminated from further consideration, since it accounted for only .7 percent of the total factor variance. Table 47 gives the percentage of the total factor variance and the percentage of total variance accounted for by each factor. Using the rotated factor matrix found in Table 48, the variable correlating most highly with each factor was identified, and these variables, NDM, YDA, YDM, LDF, SDF, LFS, and NDP, were used to construct Model 10F. Model 10F, the data for which appear in Table 49, had an  $R^2$  value of .7101 and a corrected  $R^2$  value of .6974. This model has the highest value of  $R^2$  and corrected  $R^2$  for all the process models considered so far. The comparisons are summarized in Table 50. A summary of the restricted models is given in Tables 77 and 78 in Appendix B.

Model 11. This model contains the same variables as Model 10 but has, in addition, the 4 Cromer digit variables. The data for Model 11 are presented in Table 51. It yielded an  $R^2$  value of .7665 and a corrected  $R^2$  of .7417. These  $R^2$  values are lower than those for the process and digit models 1, 4, and 9. The corrected  $R^2$  value is also lower than the corrected  $R^2$  of Model 7.

The 16 variables, omitting TSM and TSA, were subjected to a factor analysis which yielded 10 factors, of which one factor was dropped from consideration since, when it was combined the other factors accounted for only .0 percent of the total factor variance. Table 52 gives the percentage of factor variance and the percentage of total variance accounted for by each factor. The rotated factor matrix, given in Table 53, was then used to determine which variable correlated most highly with each factor. The variables identified were NWM, YDM, YDA, TDF, UDS, TDS, LDF, NWA, and NDP. These variables were then used to construct Model 11F, data for which are given in Table 54. This model had an  $R^2$  value of .7143 and a corrected  $R^2$  of .6980. These values are above the values of some of the models (Models 1F, 1FC, and 7F) and below Model 9F. These comparisons are summarized in Table 55. A summary of the restricted models for these two models is presented in Tables 79 and 80 in Appendix B.

The 24 One-Variable Models. Each of the 24 variables was used to construct a model to test the amount of the variance accounted for by that variable. The  $R^2$  values ranged from .0060 for ORD to .5254 for TSM. The findings for these 24 models are summarized in Table 56. All of the factors except ORD, SMD, and LFS accounted for a significant proportion of the variance at the .01 level.

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TABLE 47  
PROPORTION OF VARIANCE ACCOUNTED  
FOR BY THE FACTORS  
MODEL 10

Factor Number	Percentage of Total Factor Variance	Percentage of Total Variance
1	38.8	29.4
2	18.7	14.2
3	10.5	7.9
4	10.3	7.8
5	9.9	7.5
6	7.0	5.3
7	4.1	3.1

TABLE 48  
ROTATED FACTOR MATRIX  
MODEL 10

Var./Factor		1	2	3	4	5	6	7
1	NDM	.894	.189	-.037	.069	.113	.227	.144
2	YDM	.028	-.019	.781	.035	.009	.017	.002
3	NOM	.771	.137	.399	.212	.242	.188	-.104
4	NWM	.817	.241	-.026	.250	.148	.379	.105
5	NDA	.503	.479	-.201	.311	.418	.313	.138
6	YDA	.110	.809	.020	.163	.053	.063	.023
7	NOA	.522	.663	-.022	.330	.323	.030	.079
8	NWA	.508	.392	-.201	.341	.481	.318	.252
9	LDF	.138	.205	.061	.506	.088	.043	.048
10	SDF	.499	.233	.307	.236	.511	.087	.034
11	NDP	.594	.138	.028	.399	.150	.197	.491
12	LFS	.145	.030	.025	.028	.034	.397	.023

TABLE 49  
BASIC REGRESSION STATISTICS  
MODEL 10F

Variable	Type	Variables in the Equation						Sig. Level
		Regression Coefficient	Std. Error of Regression Coefficient	Partial 1	T-value with 160 Deg. Freedom	Partial F Value with 1 and 160 Deg. Freedom		
				Correlation Coefficient	Deg. Freedom	Deg. Freedom		
25	DIFF	dependent	-.254490	.035496	-.493	-7.16954	51.40226	.0000
	6	NDM	.022427	.005224	.321	4.29283	18.42843	.0000
	12	YDA	.007947	.005504	.113	1.44383	2.08464	.1507
	7	YDM	.024554	.006565	.284	3.74010	13.98831	.0003
	20	LDF	.024655	.0033370	.501	7.31635	53.52894	.0000
	21	SDF	.018769	.003656	.376	5.13315	26.34922	.0000
	24	LFS	.001749	.009278	.015	.18851	.03554	.8507
	22	NDP	.014354	.010761	.0897.	1.33390	1.77930	.1841

ANALYSIS OF VARIANCE SUMMARY TABLE				
Source of Variation	Sum of Squares	Deg. Freedom	Mean Square	
Linear Regression	1.28809	7	.18401	
Residuals from Regression	.52598	160	.00329	
Corrected Total	1.81407	167		

F-Ratio = 55.98 with 7 and 160 Deg. Freedom  
 Significance Level of F-Ratio = .0000

Correction for Mean <sup>1</sup>  
 Uncorrected Total 168

TABLE 50  
COMPARISON OF  $R^2$  FOR MODELS  
3, 3F, 3FC, 5, 5F, 6, 6F, 8, 8F, 10, AND 10F

Full Model	Restricted Model	$R^2$ Diff.	Corr. $R^2$ Diff..
Model 3	Model 10	.0209	.0277
Model 5	Model 10	.0160	.0117
Model 6	Model 10	-.0845	-.0813
Model 8	Model 10	.0159	.0152
Model 10	Model 10F	.0144	.0058
Model 3F	Model 10F	-.0475	-.0439
Model 3FC	Model 10F	-.1052	-.0997
Model 5F	Model 10F	-.0175	-.0197
Model 6F	Model 10F	-.1224	-.1199
Model 8F	Model 10F	-.0085	-.0108



**TABLE 52**  
**PROPORTION OF VARIANCE ACCOUNTED**  
**FOR BY THE FACTORS**  
**MODEL 11**

Factor Number	Percentage of Total Factor Variance	Percentage of Total Variance
1	37.8	31.7
2	13.0	10.9
3	12.4	10.4
4	9.9	8.3
5	7.7	6.4
6	7.6	6.3
7	7.1	6.0
8	3.1	2.6
9	1.5	1.2

TABLE 53  
ROTATED FACTOR MATRIX  
MODEL 11

Var./Factor		1	2	3	4	5	6	7	8	9
1	TDF	.131	-.094	.054	.792	-.052	.186	.254	-.013	.013
2	UDF	.014	.809	.089	.013	.007	-.075	.229	.118	-.138
3	TDS	.532	-.027	.024	-.223	-.184	.631	.318	.062	.046
4	UDS	.331	-.070	.138	-.068	.784	-.055	.121	.022	-.003
5	NDM	.906	-.015	.121	-.046	.302	.042	-.002	-.060	.042
6	YDM	.018	.818	-.030	-.063	-.077	.050	-.057	-.124	.125
7	NOM	.770	.457	.078	-.038	.287	.076	.065	-.025	-.149
8	NWM	.940	.021	.201	-.091	.100	.094	.140	.001	-.065
9	NDA	.729	-.087	.459	-.017	.128	.142	.185	.393	-.023
10	YDA	.186	.058	.834	-.015	.058	-.024	.101	.011	.022
11	NOA	.576	.066	.648	.139	.274	.218	.162	.102	-.076
12	NWA	.739	-.087	.392	.019	.058	.242	.184	.425	.011
13	LDF	.174	.157	.171	.139	.128	.127	.737	.038	.006
14	SDF	.530	.386	.210	.117	.274	.407	.040	.176	-.156
15	NDP	.760	.005	.131	.137	-.002	.351	.202	.006	.319
16	LFS	.199	-.048	.028	-.751	.007	.351	.107	-.024	.014



TABLE 55  
COMPARISON OF MODELS

Full Model	Restricted Model	$R^2$ Diff.	Corr. $R^2$ Diff.
Model 11	Model 10	.0420	.0385
Model 1	Model 11	.0098	.0141
Model 4	Model 11	.0198	.0147
Model 7	Model 11	-.0052	.0011
Model 9	Model 11	.0166	.0168
Model 11	Model 11F	.0522	.0437
Model 1F	Model 11F	-.0090	.0056
Model 1FC	Model 11F	-.0571	.0493
Model 7F	Model 11F	-.0648	-.0638
Model 9F	Model 11F	.0196	.0207

TABLE 56  
SUMMARY OF ONE-VARIABLE MODELS

Variable	R <sup>2</sup>	Corrected R <sup>2</sup>	F Ratio	df	Significance Level
ORD	.0060	.0000	1.00	1/166	.3198
TDF	.0498	.0441	8.71	1/166	.0036
UDF	.2073	.2026	43.42	1/166	.0000
TDS	.2920	.2878	68.48	1/166	.0000
UDS	.1513	.1462	29.59	1/166	.0000
NDM	.3858	.3821	104.25	1/166	.0000
YDM	.0767	.0712	13.79	1/166	.0000
NOM	.5123	.5094	174.38	1/166	.0000
NWM	.4841	.4810	155.77	1/166	.0000
TSM	.5254	.5225	183.77	1/166	.0000
NDA	.4100	.4065	115.37	1/166	.0000
YDA	.1323	.1271	25.31	1/166	.0000
NOA	.4511	.4529	139.22	1/166	.0000
NWA	.4160	.4125	118.24	1/166	.0000
TSA	.4715	.4683	148.07	1/166	.0000
OA	.4572	.4540	139.85	1/166	.0000
OM	.4871	.4840	157.62	1/166	.0000
DCA	.2647	.2603	59.77	1/166	.0000
DCM	.4349	.4315	127.77	1/166	.0000
LDF	.3125	.3084	75.46	1/166	.0000
SDF	.4698	.4666	147.07	1/166	.0000
NDP	.3922	.3886	107.12	1/166	.0000
SMD	.0090	.0030	1.51	1/166	.2209
LFS	.0303	.0245	5.19	1/166	.0240